

**FACULTY OF ENGINEERING**  
**B.E. (Except I.T.) III-Semester (CBCS) (Backlog) Examination, July 2021**

**Subject : Engineering Mathematics - III**

**Max. Marks: 70**

**Time: 2 hours**

**Note: Missing data, if any, may be suitably assumed.**

**PART - A**

**(5x2 = 10 Marks)**

**Answer any five questions.**

- 1 Determine 'p' such that  $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1}\left(\frac{px}{y}\right)$  is an analytic function.
- 2 Evaluate  $\oint_C \frac{e^{2z}}{(z+2)^4} dz$  where C is the circle  $|z| = 3$ .
- 3 Find the residue of  $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$  at its pole  $z = 2$
- 4 Classify singularity of the function  $f(z) = \frac{e^{1/z}}{z^2}$ .
- 5 State Dirichlet's conditions for the existence of Fourier series.
- 6 Find  $b_n$  in the Fourier series expansion of  $f(x) = 2x - x^2$  in  $(0, 3)$ .
- 7 Find the partial differential equation by eliminating the arbitrary function  $f$  from  $z = f(x^2 - y^2)$ .
- 8 Solve  $xp + yq = 3z$ .
- 9 Solve  $z = px + qy + \sqrt{1 + p^2 + q^2}$ .
- 10 Find a particular integral of the equation

$$\frac{\partial^3 z}{\partial x^3} - 3 \frac{\partial^2 z}{\partial x^2 \partial y} + 4 \frac{\partial^3 z}{\partial y^3} = e^{x+y}$$

**PART - B**

**(4x15 = 60 Marks)**

**Answer any four questions.**

- 11(a) Find the analytic function  $f(z) = u + iv$  where  $v(x, y) = e^{-x}(x \cos y + y \sin y)$ .

(b) If  $F(a) = \oint_C \frac{4z^2 + z + 5}{z - a} dz$  where C is the ellipse  $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ , then find,

- (i)  $F(3.5)$  (ii)  $F(i)$  (iii)  $F'(-1)$  (iv)  $F''(-i)$

- 2 (a) Find the Laurent's series expansion of  $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$  in the region

$3 < |z+2| < 5$ .

(b) Evaluate  $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 1}$ .

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13 Obtain half range Fourier cosine series of the function

$$f(x) = \begin{cases} kx, & 0 \leq x \leq l/2 \\ k(l-x), & l/2 \leq x \leq l \end{cases}$$

Hence find the sum  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

14 (a) Solve  $x(y-z)p + y(z-x)q = z(x-y)$ .

(b) Solve  $2xz - px^2 - 2qxy + pq = 0$  by Charpit's method.

15 Solve  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  with boundary conditions  $u(x, 0) = 3\sin\pi x$ , and  $u(0, t) = u(1, t) = 0$  where  $0 < x < 1, t > 0$ .

16 (a) Show that the bilinear transformation  $w = \frac{2z+3}{z-4}$  maps the circle  $x^2 + y^2 - 4x = 0$  into the line  $4u + 3 = 0$ .

Using Cauchy's residue theorem evaluate  $\oint_C \frac{z}{(z-1)(z-2)^2} dz$  where  $C$  is the circle  $|z-2| = \frac{1}{2}$

17 (a) Solve

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$$

(b) Solve  $z^2 = pq$ .